ABSTRACT

The aim of this study is to estimate the tail dependence between stock index returns and foreign exchange rate returns for four East Asian economies. We apply the concept of copula to model the dependence structure in the tail area between the two returns series. My major findings are that South Korea and Indonesia have much stronger lower tail dependency than right tail, indicating that the higher probability of double losses than double gains. Taiwan has symmetric tail dependence with similar upper and lower tail coefficients. In the case of the more advanced economy, there exists neither lower nor upper tail dependence.

INTRODUCTION

Understanding the dependence between risk factors is crucial in risk management and asset allocation. This study aims to examine the tail dependence between the stock index returns and exchange rate returns of four East Asian economies. Tail behavior of random variables during extreme events, such as financial crisis, can be captured via measures of tail dependence. In our case, tail dependence measures the probability that we will observe extremely large gain in the stock market, given that the local currency also has had a large appreciation against the USD. For a U.S. investor seeking international diversification, he/she will experience double large gains, one in the equity market, and the other in the currency market when translating the local currency investment into U.S. dollars. Likewise, when the stock market crashes, the foreign investor not only loses big in the stock market, but also in the currency market. Therefore, goal of risk reduction cannot be achieved due to this possibility.

The questions we endeavor to answer in this study are: 1) can investing in East Asian stock markets provide any diversification benefits? If we find positive tail dependence between the two markets, then we can claim that, for a U.S. investor, investing in the East Asian equity markets can’t provide any risk reduction benefits when it is most needed (correlation breakdown). 2) are the tail dependency similar for the countries? 3) do the tails exhibit symmetric or asymmetric dependency in that economic region? By answering these questions we hope to better understand the co-movements of equity-currency markets for the selected countries in this economic region.

Extensive research has been done on the relationship between these two markets, both theoretically and empirically. On the theoretical ground, there are two models explaining the causal relationship between the equity and currency markets: one is the “stock oriented” model of exchange rate (Branson (1983) and Frenkel (1983)) and the other is “flow-oriented” model of exchange rate (Dornbusch and...
Fisher (1980)). From the microeconomic point of view, local currency appreciation can cause exporting firms competitive disadvantage, therefore lowering their stock prices, indicating negative relationship between stock returns and exchange rate. On the other hand, importing firms can benefit from home currency appreciation, suggesting a positive relationship between these two markets. From the macroeconomic point of view, if domestic interest rate is higher relative to the rest of the world, the higher demand for home currency leads to its appreciation. In the meantime, higher interest rates also increase domestic firms borrowing cost, causing lower stock prices. This suggests a negative relationship between these two markets. Mixed results have been documented on the causal relationship between these two markets. Using the ordinary least squares (OLS) estimation, Solnik (1987) finds weak positive relationship for monthly data but negative relationship for quarterly data for eight western markets. Based on error correction model (ECM), Ajayi and Mougoue (1996) find that, in the short run, the relationship between stock prices and home currency is negative, but positive in the long run. Using a GARCH approach, Patro et al. (2002) find significant currency risk exposure in country equity index returns for 16 OECD countries. Using Granger causality test, Pan et al. (2007) study the dynamic linkages between exchange rates and stock prices for several East Asian countries. They find a significant causal relationship from exchange rates to stock prices before the 1997 Asian financial crises.

The conventional dependence measure is constructed as an average of deviations from the mean and it doesn’t distinguish between large or small realizations or between positive and negative returns. And it is based on assumptions of a linear relationship and a multivariate Gaussian distribution. Since the research of Embrechts et al. (2002) identified the limitations of correlation-based models in risk management, copula method has become more popular an approach in modeling dependence structure between financial variables. Copulas can capture dependence throughout the entire distribution of asset returns, independent of the univariate returns distribution. Not only can copulas model the degree of dependence, but also the structure of dependence. Works using copulas include Mashal and Zeevi (2002), Hu (2006), Chollete et al. (2006, 2009), and Rodriguez (2007) on the dependence structure across international equity markets, Patton (2006) on dependence structure on currency markets, and Ning (2010) on the dependence between equity-currency markets, just to name a few.

This study is similar to the previous literature in the sense that it also models dependence in international financial market returns. It is different from the existing works and contributes to the literature in the following way: first, the countries and data period are different; secondly, this paper studies the degree of tail dependence using unconditional copulas as well as conditional copulas. Our key empirical result reveals that the tail dependence coefficient is significant for the three East Asian emerging markets, and for Singapore, there isn’t enough evidence to support the existence of any tail dependency between the currency returns and stock index returns. Our findings have important implications in risk management and asset pricing. For global investors seeking to diversify their portfolio into emerging markets, ignoring the joint downside risk would underestimate the value-at-risk (VaR), which is a common market risk measure in risk management practice. Our finding should also affect the pricing of assets. As
pointed out by Phylaktis and Ravazzolo (2004), an international capital asset pricing model (ICAPM) will be mis-specified if currency risk is omitted. Poon et al. (2004) states that, tail dependence is a true measure for systematic risk in times of financial crisis and global investors should be compensated for exposure to such risk during joint market down turns.

The remainder of this paper is organized as follows. Section 2 introduces copula concepts and measure of tail dependence. Section 3 specifies the models and estimation method. In section 4, we describe the data used and present the empirical evidence of extreme co-movements. We offer concluding remarks in section 5.

COPULA CONCEPTS AND TAIL DEPENDENCE

Dependence between random variables can be modeled by copula method. In this section we introduce the general concept of copulas and some copulas used to model tail dependence in finance literature. Copulas represent a statistical tool to measure the dependence structures between financial markets, risk factors and other relevant financial variables. Copula method to model dependence is becoming more and more popular among academics and practitioners in the field of finance due to the inability of the linear correlation to handle the fat-tail problem in financial returns. There are some advantages of copula method over traditional methods: one is that copulas allow modeling nonlinear dependence structure; secondly, no assumption required regarding the marginal distributions; lastly, we can also use copulas to model tail events, which is often a paramount concern in financial risk management.

As described in Joe (1997), a copula is a multivariate distribution function that is used to bind each marginal distribution function to form the joint distribution function. Copulas parameterize the dependence between the margins, while the parameters of each marginal distribution function can be estimated separately.

SKYLAR’S THEOREMS AND COUPLAS

The theorem central to the theory of copulas is called Sklar’s theorem. In 1959, A. Sklar (1959) created a new class of functions now known as copulas, which couple a joint distribution function to its univariate marginals. We will present this theorem mainly by following Nelson (1999).

Sklar’s Theorem (Sklar 1959).Let H be a joint distribution function with marginals F and G. then there exists a copula C such that for all x, y in \( \mathbb{R} \),

\[
H(x, y) = C(F(x), G(y))
\]  

(1)

If F and G are continuous, then C is unique; otherwise, C is uniquely determined on \( \text{Ran} F \times \text{Ran} G \). Conversely, if C is a copula and F and G are distribution functions, then the function H defined by the above equation is a joint distribution function with marginals F and G.

Definition 1: A two-dimensional copula is a function \( C: [0,1]^2 \rightarrow [0,1] \) which satisfies the following properties:
(a) Grounded: for every \(u, v\) in \([0,1]\), \(C(u, 0) = 0 = C(0, v)\):

(b) \[ C(u, 1) = u \text{ and } C(1, v) = v \text{ for all } (u, v) \in [0,1]^2; \]

(c) 2-increasing: for every \(u_1, u_2, v_1, v_2\) in \([0,1]\) such that \(u_1 \leq u_2\) and \(v_1 \leq v_2\), \(C(u_1, v_2) - C(u_2, v_2) - C(u_1, v_1) + C(u_2, v_1) \geq 0\).

Hence, any bivariate distribution function whose margins are standard uniform distributions is a copula. From the definition, we know that copulas are joint distribution functions of standard uniform random variables:

\[ C(u, v) = \Pr(U_1 \leq u, U_2 \leq v) \]

For a more detailed treatment of copulas, the reader can refer to Joe (1997) and Nelson (1999). For an overview of copula applications in finance, see Cherubini et al. (2004) and Patton (2009) for copula applications in financial time series.

*Measure of tail dependence*

Tail dependence refers to the amount of dependence in the tails of a bivariate distribution or alternatively the dependence in the corner of the lower-left quadrant or upper-right quadrant of a bivariate distribution. Tail dependence between two random variables is a copula property and hence the amount of tail dependence is invariant under strictly increasing transformations of \(X\) and \(Y\). For two random variables \(X\) and \(Y\) with marginal distributions \(F_X(x)\) and \(F_Y(y)\), the upper tail dependence is

\[
\lambda_r = \lim_{u \to 1^-} \Pr[F_Y(y) \geq u | F_X(x) \geq u] = \lim_{u \to 1^-} \frac{1 - 2u + C(u, u)}{1 - u} \tag{2}
\]

and the lower tail dependence is

\[
\lambda_l = \lim_{u \to 0^+} \Pr[F_Y(y) \leq u | F_X(x) \leq u] = \lim_{u \to 0^+} \frac{C(u, u)}{u} \tag{3}
\]

where \(\lambda_r\) and \(\lambda_l\) \in [0,1]. Positive \(\lambda_l\) or \(\lambda_r\) indicates that \(X\) and \(Y\) are to be tail dependent. If the tail dependence coefficient is zero, the variables are asymptotically independent. However, tail independent does not mean that the variables are independent. The copulas with different tail dependence structure applied in this study are introduced in the next section.

**ESTIMATION METHOD AND MODEL SPECIFICATION**

Generally speaking, there are two approaches to estimate copula models, one is the one-stage full maximum likelihood estimation method, and the other is the two-stage inference functions for margins (IFM) method proposed by Joe and Xu (1996). The one-stage approach jointly estimates the parameters of the marginal models and parameters of the copula models simultaneously. Given the large number of parameters, this method can be computationally intensive and make the numerical maximization of the log likelihood function difficult. Therefore, in practice, the two-stage IFM method is preferred due to its computational tractability. Under the IFM approach, the first step models the marginal models, either parametrically or non-parametrically. If estimation is done non-parametrically, then the method is a semi-
parametric two-step estimation method, also known as Canonical Maximum Likelihood, or CML method. Copula parameters are estimated in the second step. For more details on this estimation method, interested reader can refer to Cherubini et al. (2004). Joe (1997) points out that the IFM is a highly efficient method, and he proves that the IFM estimator is consistent and asymptotically normal under standard conditions.

THE MARGINAL MODELS

We model the marginal distributions parametrically using GARCH type models. In the finance literature, a very common approach to model time series is the generalized autoregressive conditional heteroskedasticity (GARCH) model. In particular, we filter the raw returns data with a AR(k)-GARCH(p, q) or AR(k)-t-GARCH(p, q) type models. This type of models has been used in Bollerslev (1987), Patton (2006a), and Ning (2010) among others. The marginal model is specified as follows:

\[ \eta_{i,t} = C_i + \sum_k A_{i,k} \times \eta_{i,t-k} + \varepsilon_{i,t} \]  
\[ \sigma^2_{i,t} = \text{Arch}_i + \sum_p \text{Garch}(p) \times \sigma^2_{i,t-p} + \sum_q \text{Arch}(q) \times \varepsilon^2_{i,t-q} \]

where \( \eta_{i,t} \) is the returns for country \( i \) at time \( t \), \( \sigma^2_{i,t} \) is the variance of \( \varepsilon_{i,t} \) term in the mean equation (equation (4)). Estimation results of the marginal model are discussed in subsection 4.2.

Static copula models

Student’s t-copula

The Student’s t-copula is based on the multivariate t distribution, in the same way as the Gaussian copula is derived from the multivariate normal distribution. The copula of the bivariate Student’s t-distribution with a degree of freedom of \( v \) and correlation \( \rho \) is

\[ C_{p,v}(u,v) = \int_{-\infty}^{u} \int_{-\infty}^{v} \frac{1}{2\pi \sqrt{1-\rho^2}} \left\{ 1 - \frac{(u^2 + v^2 - 2uv\rho)}{(v^2 - 2uv\rho)} \right\}^{-\nu/2} \, ds \, dt \]  

As the value of \( v \) increases, say \( v = 100 \), it approximates a Gaussian distribution. The bivariate Student’s t-copula exhibits symmetric tail dependence and has the tail independence Gaussian copula as a special case.

CLAYTON COUPULA

Clayton copula belongs to the Archimedean Copula family and is known to have tail dependence. The bivariate Clayton copula can be written as the following

\[ C_{\hat{\theta}}(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta} \]  

where \( 0 < \theta < \infty \) is a parameter controlling the dependence, \( \theta \to 0^+ \) implies independence, and \( \theta \to \infty \) implies perfect dependence. \( u \) and \( v \) are standard uniformly distributed i.i.d.s. Clayton copula can be used to describe lower (left) tail dependence and no upper (right) tail dependence.

Symmetrized Joe-Clayton copula (SJC)
Symmetrized Joe-Clayton (SJC) copula allows both upper and lower tail dependence and symmetric dependence as a special case. The SJC copula is a modified version of the Joe-Clayton copula (Joe 1997), as proposed by Patton (2006a) and it is defined as follows.

\[ C_{SJC}(u, v|\lambda_r, \lambda_l) = 0.5 \cdot (C_{JC}(u, v|\lambda_r, \lambda_l) + C_{JC}(1 - u, 1 - v|\lambda_r, \lambda_l) + u + v - 1) \]  
\[ C_{JC}(u, v|\lambda_r, \lambda_l) = 1 - \left(1 - \left(1 - (1 - u)^{\lambda_r}\right)\right)^{-\gamma} + \left(1 - \left(1 - v\right)^{\lambda_l}\right)^{-\gamma} - 1)^{-1/\gamma}\right)^{-1/k} \]

where \( C_{JC}(u, v|\lambda_r, \lambda_l) \) is the Joe-Clayton copula defined as follows.

\[ \Lambda denotes the logistic transformation to keep the tail dependency parameter of the SJC copula in [0,1] and it is defined as \( \Lambda(x) = \left(1 + e^{-x}\right)^{-1}. \]

The dynamic copula model contains an autoregressive term designed to capture persistence in dependence and a forcing variable which is the mean absolute difference between \( u \) and \( v \). The forcing variable is positive when the two probability integral transforms are on the opposite side of the extremes of the joint distribution and close to zero when they are on the same side of the extremes.

**DATA AND EMPIRICAL RESULTS**

**DATA**

The dataset used in this chapter consists of daily closing stock index returns and foreign exchange rate movements for four East Asian economies (Indonesia, South Korea, Singapore, and Taiwan). The stock indices are the Jakarta SE Composite Index of Indonesia, the Korea Stock Exchange Stock Price Index (KOSPI), The Strait Times Stock Exchange of Singapore, and the Taiwan Stock Exchange Capitalization Weighted Index. The corresponding exchange rates are Indonesia Rupiah (US$/IDR), Korean Won (US$/KRW), Singapore dollar (US$/SGD), and Taiwan dollar (US$/TWD). The dataset has different starting dates, but all end on June 4, 2010.

Table 1 presents summary statistics of the continuously compounded stock index returns and currency movements for each country, and all returns are in percentage terms. As manifested from the table, East Asian equity markets did provide higher returns (with the exception of Taiwan for the sample period), at the expense of higher risk, as measured by the sample standard deviation. Generally, the standard deviation of equity returns is higher than that of currency movements, with the exception of...
Indonesia. The nonzero skewness measure and excess kurtosis all point to the non-normality of the returns. Our Jarque-Bera tests further confirm the non-normality of the returns data (not reported in the table).

Three traditional correlation measures are presented in TABLE 2: Pearson’s linear correlation, the Kendall’s tau, and Spearman’s rho rank correlation coefficients. To see the dependence structure from our data, we also calculate the empirical copula for the country pairs (see Knight et al. (2005)). We first rank the pair of the returns series in ascending order and each series is divided evenly into 10 bins. Bin one includes the observations with the lowest values (in their lowest 10th percentile) and Bin ten includes observations in the top 10th percentile. The resulting table will show us how the two returns series are associated with each other. If the two series are perfectly positively related, we expect all the observations lie on the major diagonal. If they are negatively related, most observations should lie in the cells on the diagonal connecting the lower-left corner and upper-right corner. If there is positive right tail dependence, the number of observations in cell (10, 10) would be larger. We would expect a large number in cell (1, 1) if there exists positive left tail dependence. The empirical copula frequency counts for four country pairs are presented in TABLE 3. Comparing cell(1,1) and cell(10,10) of all country pairs, we observe that both upper tail and lower tail dependence are present for the sample period with a higher percentage of observations lying in the lower tail area.

RESULTS OF THE GARCH MODELS

The parameter estimates and standard errors for marginal distribution models are reported in TABLE 4 and TABLE 5. Only the highly significant (with 5% significance level at least) autoregressive terms and GARCH terms are reported in the table. For most of the return series, GARCH(1,1) is sufficient to model the conditional heteroskedasticity, but some require higher Arch/Garch terms. This is shown by significant Arch2, Arch3, and Garch8 terms for Indonesia currency returns. A Gaussian conditional probability is sufficient for most of the marginal models, except for the stock index returns of South Korea. In the next subsection, we discuss the results of the copula models.

Empirical results of the static copula models

Parameter estimates of the SJC copula, Student’s t-copula, and Clayton copula models are presented in Table 7. We observe significant estimates of the parameters of the lower tail dependence as well as upper tail dependence for the three emerging markets, namely, Indonesia pair, South Korea pair and Taiwan pair. For Singapore, there is no evidence on tail dependence, i.e. the tail dependence parameters are not significant at either tail. Whereas Indonesia pair and South Korea pair have asymmetric tail dependence, Taiwan pair has symmetric tail dependence measure as the estimated parameters are not significantly different. The estimated degree of freedom (v) of the Student’s t-copula ranges from 6.74 (Korea) to 16.06 (Taiwan), indicating bivariate non-normality between the returns distributions of the two markets for the countries under study. This further confirms that linear correlation coefficient as a measure of dependence between financial returns can give misleading results. For
Singapore, even though not enough evidence to show that there exist extreme co-movement between the equity-currency markets, but the estimated degree of freedom parameter of 12.12, indicating that bivariate normal distribution is not reasonably a good assumption in modeling the dependence between the two returns series.

Empirical results of the dynamic copula models

Next we look at the dynamics of the tail dependence measures. Since the static copula results indicate no tail dependence in Singapore financial markets, here we focus on the three emerging markets. We apply Patton (2006) time-varying SJC copula to examine the conditional bivariate distribution of the returns series for Indonesia, South Korea, and Taiwan. TABLE 8 reports the parameter estimates along with the static SJC copula results for convenience. Our empirical results show that the autoregressive term for both tails of Korea pair (lower tail $\beta = 0.8947$, upper tail $\beta = 0.9737$), upper tail of Indonesia pair ($\beta = 0.9210$), and lower tail for Taiwan pair ($\beta = 0.9079$), is significant, indicating the high persistence in the dependence level. The parameters for the lower tail dependence coefficient of Indonesia pair are not significantly different from zero, indicating that there is no significant change in the degree of the tail dependence.

To illustrate the evolving time path of the degree of tail dependence coefficients, in Figures 1, 2, and 3, we plot the conditional upper and lower tail dependence implied by the time-varying SJC copula model. In the figures, we also plot the time-varying difference between lower tail and upper tail coefficients, as calculated by $\lambda_l - \lambda_r$. Under symmetry, this difference should be zero. From the bottom plot of Figures 1 and 2 we note that conditional lower tail dependence is greater than conditional upper tail dependence almost all the time for Indonesia pair and South Korea pair, supporting our conclusion of asymmetry in tail dependencies for these two pairs. In the case of Taiwan (Figure 3), the difference between the lower tail coefficient and upper tail coefficient fluctuates around zero, indicating that it is not significantly different in the lower and upper tail parameter values, as we have concluded earlier based on unconditional copula results.

We can compare the relative performance of the competing copula models using Akaike’s information criterion (AIC). For the three pairs, we find a reduction of the AIC in the time-varying SJC model (Korea pair decreased the most and Taiwan pair decreased the least), indicating the dynamic copula model performs better than their static counterpart.

CONCLUSION

In this paper, we examine the degree of dependence at the extremes of the bivariate distribution between the stock index returns and foreign exchange fluctuations in four East Asian economies via copula methods. Using static copula models, our major findings are the following: 1) for the more advanced economy, namely Singapore, there is no evidence of tail dependence between the two returns series; 2) for the three emerging markets, Indonesia and South Korea have significantly higher left tail dependency than right tail dependency, thus asymmetric tail dependencies. For Taiwan, the tail
dependence is significant and similar between the lower and upper tail, suggesting symmetric tail dependence behavior.

We also employ Patton (2006) conditional SJC copula model to examine the dynamics of tail dependence coefficients between stock index returns and foreign exchange rate fluctuations for the three emerging markets. The empirical results show that the autoregressive term for both tails of South Korea pair, upper tail of Indonesia pair, and lower tail for Taiwan pair is significant, indicating the high persistence in the dependence level. Using graphical analysis, the conditional lower tail dependence is greater than conditional upper tail dependence almost all the time for Indonesia pair and South Korea pair, supporting the conclusion of asymmetry in tail dependencies for these two countries.

Our empirical findings have important finance implications in risk management and asset pricing. For investors seeking to diversify their portfolio into emerging financial markets, ignoring the joint downside risk would underestimate the value-at-risk (VaR), which is a common risk measure in risk management. Tail dependence serves as a true measure for systematic risk in times of financial crisis and global investors should be compensated for exposure to such risk during joint market downturns. These results can provide important guidance for investors who consider international diversification into this economic region. For international investors seeking diversification into Indonesia and South Korea stock markets, it is more likely for them experiencing extreme double losses (one in the stock market and the other in the currency market when translating into home currency returns) than extreme double gains, therefore hedging equity investments with currency derivatives is highly recommended. For investments made in the advanced market, currency hedging does not seem quite necessary.

ENDNOTES
1. I am grateful for helpful suggestions from participants at the 38th Annual Conference of the Eastern Economic Association, Boston, MA, March 2012.
2. The tables and figures are available from the author upon request.

REFERENCES


